

A NEGLECTED VERSION OF THE ANECDOTE
ABOUT PYTHAGORAS'S HAMMER EXPERIMENT,

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There is a widely spread story about Pythagoras, which describes how by chance he found a means of demonstrating the simple numeric proportions that govern the musical intervals of the fourth, the fifth, and the octave. Our oldest source for this story is Nicomachus from Gerasa (2nd cent. A.D.), and since his time the story has been told again and again, by Greeks and Romans alike.¹ It was one of the few 'facts' about Pythagoras which were remembered in the Middle Ages, cf. the lovely woodcuts in Franchino Gafori's *Theorica Musicae* from 1492 which illustrate a number of acoustic experiments, including Pythagoras's hammers.²

According to Nicomachus, Pythagoras one day passed by a smithy (παρά τι χαλκοτυπεῖον) and heard the sound of hammers against iron on an anvil (ἐπήκουσε ραιστήρων σίδηρον ἐπ' ἄκμονι ραίωντων). In their different sounds he recognized the octave, the fifth, and the fourth. Full of joy he ran into the smithy (τὸ χαλκεῖον) and tried to find out what produced these intervals. A number of experiments made on the spot made him realize that the difference in the sounds was produced by the weight of the hammers (παρά τὸν ἐν τοῖς ραιστήρσιν ὄγκον). It was not produced by the strength of the various blows (παρά τὴν τῶν ραίωντων βίαν), nor by the shapes of the hammers (τὰ σχήματα τῶν σφυρῶν), nor by any change of the

1. Of Nicomachus's musical works, only the short *Enchiridion* ('Ἀρμονικὸν ἐγγχειρίδιον) has been preserved, see C. Jan, *Musici Scriptores Graeci*, Lips. 1895, pp. 209-82. The anecdote (*Enchiridion* ch.VI, pp. 245 sqq. Jan) must also have been included in his lost *Εἰσαγωγὴ μουσική*; for it is told by Boethius in *De institutione musica* I,10. The dependency of Boethius on Nicomachus's lost treatise has most recently been demonstrated by Calvin Bower in his article on 'Boethius and Nichomachus: An Essay Concerning the Sources of *De institutione musica*' (*Vivarium* 16, 1978, pp. 1-45).

2. See illustration, opposite.

position of the iron (ἡ τοῦ ἐλαυνομένου σιδήρου μετάθεσις). In the following, Nicomachus tells how Pythagoras now took four strings of identical length and structure, weighed them down with weights proportional to the weights of the hammers in the smithy - and was thus able to produce intervals of a fourth, a fifth, and an octave, and to demonstrate the simple numeric relations that existed between these basic intervals. Nicomachus then goes on with a detailed description of the stringed instrument which Pythagoras constructed for purposes of demonstration. With this instrument Pythagoras was able to check a series of other acoustic experiments - by means of blows on pots (λεκίδων κροῦσις), of flutes, monochords, triangular harps etc. - all of which experiments led him to observe the same simple numeric relations.

Any physicist can tell us why the story about Pythagoras and the Hammers cannot possibly be true. The physical truth of the story was believed until quite recently - 1634, if I am not mistaken. Now it is generally explained as a legend based on ancient - and probably sound - information on a number of Pythagorean experiments. One such experiment is mentioned in a scholion to Plato, Phaedon 108D, where Aristoxenos is quoted for a story about the early Pythagorean Hippasos, who is said to have produced the fourth, the fifth, and the octave by means of four brazen δίσκοι. The disks which Hippasos used had all the same diameter, but differed in thickness.

In 1847 A.J.H. Vincent published five extracts from the musical treatise in the Paris manuscript *Ancien fonds grec 360*, the so-called Hagiopolites.³ One of these texts ("Frag-

3. Notices et extraits des manuscrits de la bibliothèque du roi et autres bibliothèques, XVI,2, Paris 1847, pp. 259-281. For the Hagiopolites, see Christian Hannick, Byzantinische Musik (in Herbert Hunger, Die hochsprachliche profane Literatur der Byzantiner, Bd. 2, München 1978 = Byzantinisches Handbuch im Rahmen des Handbuchs der Altertumswissenschaft, 5. Teil, 2. Band), pp. 200-01 and my forthcoming article in Texte und Untersuchungen zur Geschichte der Altchristlichen Literatur Bd. 125 ("The Manuscript Tradition of the Hagiopolites: A Preliminary Investigation on Ancien Fonds Grec 360 and Its Sources").

ment V", Vincent pp. 266-73) contains the Pythagoras anecdote in a form which differs considerably from all other known sources. It runs as follows (Hagiop. fol. 234v-235r):

‘Ιστέον οὖν ὡς μὲν λόγος ἀρχαῖος τῶν θύραθεν· ὁ παρ’ ἑλλησι θρηλλούμενος πυθαγόρας παρὰ τινι χαλκείῳ πολιτικῷ καθεζόμενος· καὶ διαφόρων ἤχων ἐξ αὐτοῦ ἀκούων· καὶ ταῦτα μιᾷς ὕλης οὐσης, τῆς χαλκευομένης· καὶ τοῦ αὐτοῦ καὶ ἐνὸς σκεύους τοῦ χαλκεύοντος· καὶ τοῦ αὐτοῦ ἄκμωνος ἐν ᾧ περ ἡλαύνοντο τὰ χαλκευόμενα· σκοπὸν ἔθετο τὴν τῶν ἀποτελουμένων ἤχων διαφορὰν ὅθεν γίνεται καταλαβεῖν· καὶ δὴ πολλὰ σκοπήσας καὶ ἐρευνήσας· τέλος πρὸς τὰς σφαίρας ἐνέσκηψεν· ἃς καὶ σταθμώσας, καὶ εὐρών τὴν μὲν, βαρυτέραν· τὴν δὲ, κουφοτέραν, ἔγνω ἐντεῦθεν προῖεσθαι τὸ τῶν ἤχων διάφορον· καὶ ἀναλόγως τὴν τε κουφότητα τῶν φωνῶν ἡγουν τῶν σφαιρῶν ἀντιπαθεῖν βαρύτητι, καὶ τὰ ἀπηχήματα διάφορα γεγενῆσθαι. Ἐξ αὐτοῦ καὶ αὐτὸς παρορμηθεὶς, κατεσκεύασεν ἀπὸ χορδῶν τεσσάρων καὶ μόνον ὄργανον, ὃ κέκληκε μουσικὴν· εἴτα ἀνεβίβασεν αὐτὸ εἰς ἑπτὰ χορδὰς· καθὼς ὁ πυθαγορικὸς φιλόλαος, ἐν τινι πονήματι αὐτοῦ· πρὸς τινα γυναῖκα πυθαγορεῖαν ἐκτιθέμενος γράφει· περὶ τῆς ἁρμονικῆς φιλοσοφίας, οὕτω φάσκων· ἁρμονίας μεγέθους συλλαβῆς δι’ ὅξεϊα μείζων τὰς συλλαβάς ἐπέιγῃ...⁴

In this version we again find the smithy, the different sounds coming from it, the metal (ἡ ὕλη ἡ χαλκευομένη), and the anvil. But in the Hagiopolites version there is only one hammer (τὸ σκεῦος τὸ χαλκεύον), whereas several pieces of metalware are being forged (τὰ χαλκευόμενα). Pythagoras makes, as usual, many experiments and finds out in the end that the difference of sound (higher or lower) was due to the different weight of τῶν σφαιρῶν.

Now, the word σφαῖραι comes so close to the σφῦραι of the standard version that it suggests, at first, that τὰς

4. The corrupt Philolaos quotation (= Philolaos fr. B 6) is preserved in Stobaios Ecl. I, 21 7d (p. 188, 14) and in ch. IX of Nicomachus's Enchiridion (Jan. pp. 252-54). According to Walter Burkert, *Love and Science in Ancient Pythagoreanism*, Cambridge Mass. 1972, p. 241 note 11, the Hagiopolites "evidently" quotes from Nicomachus. This is less evident to me than it was to Burkert.

σφαίρας is nothing but a scribal error for τὰς σφύρας (the hammers). However, the Hagiopolites says clearly enough that there was only one hammer - or, at least, one type or size of hammer (τοῦ αὐτοῦ καὶ ἐνὸς σκεύους τοῦ χαλκεύοντος) - and the plural (τὰς σφαίρας) for that simple reason makes this first explanation impossible. The only interpretation which does make sense, is to take αἱ σφαῖραι to be synonymous with τὰ χαλκευόμενα. The smiths were obviously working on some σφαῖραι of different size or different weight. Exactly what does σφαῖρα mean in this context? Hardly any of the meanings listed in LSJ ("ball, globe, sphere, pill"). Let us postpone the question for a moment and look instead for eventual traces of σφαῖραι in connection with Pythagorean acoustical experiments.

In the Harmonics of Ptolemy (2nd cent. A.D.), the description of the Monochord (the demonstration instrument) is introduced by a short description of some imperfect methods of demonstrating the numeric relations between musical intervals. Having described αὐλοί, σύριγγες, and strings with attached weights, Ptolemy continues with sounds produced διὰ τῶν ἀνισοβαρῶν σφυρῶν ἢ δίσκων (by σφῦραι or disks of different weights), and from empty or filled vessels.⁵ In his critical apparatus to this passage Düring mentions a variant reading σφαιρῶν (for σφυρῶν), found in the text of V (Vatican gr. 192, 13th-14th cent.) and as a γρ-reading in G (Vatican gr. 198, 14th cent.). Now, the manuscript G is the best representative for the edition which Nicephoras Gregoras made in the 14th century⁶, and V seems to have a special position within the m-family⁷; so the testimony of these two sources should not be too readily disregarded. Besides, σφαιρῶν is an obvious *lectio difficilior* which a scribe might easily correct into the traditional hammers (σφυρῶν). The reading

5. Die Harmonielehre des Klaudios Ptolemaios, herausgegeben von Ingemar Düring, Göteborg 1930 (= Göteborgs högskolas årsskrift XXXVI.1930:1), pp. 16-17.

6. "möglicherweise das Original Exemplar der Redaktion des Gregoras oder eine Abschrift davon" (Düring, Die Harmonielehre... p.LXII).

7. Cf. Düring's *stemma codicum*, ibid. p. LXIX, and his description of m, pp. XLVIII-LIV).

of VG^{ΥΡ} is further supported by a scholion (probably from G, cf. Düring p. C) which concerns Ptolemy's description of the bars of the monochord (the μαγάδες)⁸. These Magades are described by Ptolemy as being πανταχόθεν ἴσαι τε καὶ ὅμοιαι, so that their surfaces are as round (σφαιρικός) as possible. About the use of the adjective σφαιρικός (for the surface of a cylindrical body), the scholiast says: "*Spherical* is improperly used for a body which is oblong and rounded, exactly as he has just used σφαιρῶν and δίσκων about the same thing" (ἰστέον ὅτι κατὰ παράχρησιν σφαιρικὸν σῶμα λέγει καὶ τὸ ἐπιμυκὲς μὲν, τετορευμένον δέ, ὡς καὶ ἀνώτερον σφαιρῶν καὶ δίσκων εἶπεν ἐπὶ τοῦ αὐτοῦ σημαινομένου). The scholiast - in all probability Gregoras himself - thus suggests that σφαῖραι and δίσκοι are synonyms. In this connection it is to be noticed that in the text which we have just quoted from Ptolemy (p. 17,16-18 Düring), the two words are coupled not with καὶ but with ἤ; this conjunction can hardly go with hammers and disks, but would be appropriate to connect a pair of synonyms for round objects of some kind.

Ptolemy's Harmonics were commented by Porphyry in the 3rd century.⁹ In his commentaries (pp. 120 and 121, Düring) we find the decisive proof that Ptolemy spoke of σφαῖραι, not of σφύραι. In the preceding (p. 119,12sqg.) Porphyry has mentioned experiments with flutes, strings with attached weights, and empty or filled vessels. He then continues (p. 120,13-15): ὡσαύτως δὲ καὶ ἐάν τις δίσκους χαλκοῦς ποιήσας διπλασιάσῃ θατέρου τὸν ἕτερον, συμφωνοῦσι κρουόμενα διὰ πασῶν. The context leaves no doubt that Porphyry's brazen disks correspond to the "spheres or disks" of Ptolemy. At the end of this passage Porphyry gives a paraphrase of Ptolemy's reasoning about the insufficiency of such acoustical experiments. I quote Ptolemy's text and Porphyry's paraphrase:

8. Düring, Die Harmonielehre, p. 17,27 sqq.

9. Porphyrios Kommentar zur Harmonielehre des Ptolemaios, herausgegeben von Ingemar Düring. Göteborg 1932 (= Göteborgs högskolas årsskrift XXXVIII. 1932:2).

PTOLEMY (p.17,16-20):

τὰ παραπλήσια δὲ συμβαίνει
κάπὶ τῶν κατὰ σύγκρουσιν γινο-
μένων ψόφων,
ὁποίους ἐπιννοοῦσι τοὺς διὰ
τῶν ἀνισοβαρῶν

σφυρῶν ἢ δίσκων
καὶ τοὺς ἀπὸ τῶν τρυβλίων κε-
νῶν τε καὶ πεπληρωμένων,
ἐργώδους ὄντος πάνυ τοῦ τη-
ρεῖν ἐν ἅπασι τούτοις
τὴν ἐν ταῖς ὕλαις καὶ τοῖς
σχήμασιν ἀπαραλλαξίαν.

PORPHYRY (p.121,10-14):

τὰ παραπλήσια δὲ συμβαίνει
καὶ ἐπὶ τῶν γινομένων ψόφων
κατὰ σύγκρουσιν
σφαιρῶν ναστῶν ἢ δίσκων κοίλων
ἢ ἀγγείων ἴσων καὶ ὁμοίων κενῶν
τε καὶ λαμβανόντων ὕδωρ,
δυσχεροῦς ὄντος πάνυ τοῦ τηρεῖν
ἐν ἅπασι τούτοις
καὶ τὸ ἐν ταῖς ὕλαις καὶ τοῖς
σχήμασιν αὐτῶν ἀδιάφορον.

It is quite clear that the text which Porphyry commented upon dealt with "spheres" and not with "hammers". But it seems equally clear that Porphyry did not quite understand this text. As we have already seen, the two expressions used by Ptolemy must be synonyms, relating to the same physical object. But Porphyry's use of adjectives ("*solid* spheres or *hollow* disks") makes sense only if he took the two words to refer to different objects. The inference is that he did not know the word σφαῖρα as a synonym to δίσκος.

We have now met a number of different objects associated with Pythagorean acoustical experiments: χαλκοῖ δίσκοι (Hippasos of Megapont, according to Schol. in Plat. Phaedon 108D), σφαῖραι ἢ δίσκοι (Ptolemy), σφαῖραι ναστοὶ ἢ δίσκοι κοῖλοι (Porphyry), σφαῖραι (Hagiopolites), σφῦραι (Nicomachus; most Ptolemy MSS), ραισιτήρες (Nicomachus). We may combine these traditions into a coherent though hypothetical description of how Hippasos's brazen disks were gradually transformed into Pythagoras's hammers:

At first, the story was told about Hippasos and his experiments with some brazen objects that were once called σφαῖραι, but by the scholiast (or his source) were referred to as δίσκοι. At some later time the description of these experiments was converted into an anecdote about Pythagoras; the

metalware objects were still called σφαῖραι. Still later, the anecdote was rewritten, from a source in which σφαῖραι had become σφῦραι by mistake. Once the hammers had been introduced, there was a possibility to use another word for them, δαισιήρες. This successive transformation of the story had produced the famous anecdote about Pythagoras and the Hammers already in the 2nd century A.D. (the date of Nicomachus from Gerasa), though Ptolemy in the same century still had access to a σφαῖρα-version and knew that the word in this context was a synonym to δόκοι. In the following century, Porphyry no longer knew its exact meaning, but had to rely on his understanding of the context in Ptolemy.

Evidently, this reconstruction implies a considerable age for the version of the Hagiopolites.¹⁰ It furthermore implies that the word σφαῖρα once denoted some kind of object which could be manufactured in a smithy, but that later on this meaning of the word became obsolete. Now, if we combine an idea of Burkert's with a couple of places in Athenaeus's *Deipnosophists* and Plato's *Republic*, I think that a case can be made for taking these objects to be some kind of vessels or bowls.

Let us begin with Burkert. In his treatment of the Pythagoras anecdote¹¹, he concludes that "the claim that Pythagoras discovered the basic law of acoustics in a smithy is a rationalization - physically false - of the tradition that Pythagoras knew the secret of magical music which was discovered by the mythical blacksmiths."¹² This explanation leaves no room for Hippasos's four brazen disks. To Burkert, they belong to the same tradition as the story about Lasos's experiments with vessels filled in varying degrees - going back, ultimately, to a tradition "that Lasos and Hippasus, in study-

10. According to Vincent, op.cit. p.267, its author "doit être antérieur, non seulement à Aristoxène, mais peut-être même à Aristote; et il se trouverait être ainsi *le plus ancien écrivain sur la musique, dont les œuvres ne sont pas entièrement perdues.*" (My Italics).

11. Burkert, op.cit. (cf. note 4), pp.375 sqq.

12. ibid. p. 377. The 'mythical blacksmiths' are the Idaean Dactyls.

ing phenomena of resonance, learned the numerical laws, and that a subsequent restatement in somewhat cruder form transformed their action to one of striking the vessels."¹³ The resonance which Burkert speaks of here, is the one which in Greek theatres was produced by empty vessels, ἡχεῖα.

In Book XI of the *Deipnosophists* - which deals profusely with different kinds of cups, arranged in alphabetical order - Athenaeus quotes Asclepiades from Myrlea (1st cent. B.C.) for the following remarks on Nestor's famous cup: "The ancients, who were the first to ordain for men the things pertaining to civilized life, being convinced that the universe is spherical in shape (τὸν κόσμον εἶναι σφαιροειδῆ)... thought it was only right to make the things pertaining to their own food like the element which encompasses the earth, according to the shape it seemed to have (τῷ περιέχοντι κατὰ τὴν ἰδέαν τοῦ σχήματος ἀφομοιοῦν). Hence they made a table round (τὴν τράπεζαν κυκλοειδῆ κατεσκευάσαντο).... Hence, too, the cup, which contains liquid food, they made circular in imitation of the universe (καὶ τὸ ποτήριον οὖν τὸ δεχόμενον τὴν ὑγρὰν τροφήν κυκλοτερές ἐποίησαν κατὰ μίμημα τοῦ κόσμου)."¹⁴ Nestor's cup, as Asclepiades points out, is a particularly good example of this; for its ornamental studs (the golden ἥλοι of Δ 633) are set like stars on the firmament - in other words: Nestor's star-spangled cup imitates the celestial sphere with its fixed stars.

Older - and much more interesting than this "curious passage, wherein philology distorted by allegory and astrology is seen at its worst"¹⁵ - is Plato's Spindle of Necessity, the model of the universe described in the Myth of Er near the end of Book X of the *Republic* (616C sqq.). For although this description contains no mention at all of any σφαῖραι, the eight whorls (σφόνδυλοι) of the spindle clearly refer to the planetary and celestial spheres.¹⁶ The way in which these whorls are fit-

13. *ibid.*, p. 377-78.

14. Athenaeus, *Deipnosophistae* XI, 489c-d, translated by C.B. Gulick (Loeb).

15. Gulick, vol.V, p. 170, note a.

16. The Harmony of the Spheres is alluded to in 617B.

ted into one another is compared to a set or nest of bowls (κάδοι), their rims forming a continuous flat surface. In his description, Plato repeatedly refers to objects well-known. Thus, the whorl is said to be shaped like an ordinary whorl (τὸ μὲν σχῆμα οἷαπερ ἡ τοῦ ἐνθάδε), and in the comparison between σφόνδυλοι and κάδοι Plato's use of the definite article shows that he is thinking of some objects which he and his readers know from their daily life: καθάπερ οἱ κάδοι οἱ εἰς ἀλλήλους ἀρμόττοντες.

The celestial sphere(s) which Er and Asclepiades had in mind were compared to drinking vessels, not to balls. With Burkert's explanation of the nature of the acoustical experiments carried out by Lasos and Hippasos we once again encounter vessels, and Hippasos's δίσκοι are to be understood as a transformation of these. Σφαῖρα as a name for some vessel is by no means stranger than the English 'bowl'. Apparently, this meaning later on fell out of use and was forgotten, so that nobody any longer understood the metaphor 'sphere' for the firmament. For this is where our investigation ends: Isn't it more likely that the celestial sphere was named after its being similar to a bowl than that these bowls got their name from their likeness to the firmament?